Colour-Kinematics Duality @ Five-Loop

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Plan

- Motivation
- What's Colour-Kinematics duality?
- Five-Loop Construction
- Conclusion

Background

There has been tremendous progress on Scattering Amplitudes in Past years.

In particular, in large N_c limit (i.e. planar limit), strong coupling or even non-perturbative results can be obtained, because of AdS/CFT correspondence and Integrability.

Background

On the other hand, much less is understood in the NON-Planar sector.

For QCD ($N_c = 3$) and for Gravity theories, non-planar contribution is always important.

⇨ A major challenge:

to go beyond planar limit!

Background

Colour-Kinematics Duality [Bern, Carrasco, Johansson]

put planar and non-planar parts on the same footing.

⇨ get non-planar parts "for free".

Besides, it provides an extremely efficient way to construct the complete integrand in a very compact form.

Background

We will focus on the Integrand.

For evaluating integrals, see Henn's talk.

Why Feynman Diagram is NOT enough?

→ works in principle, but the complexity grows extremely fast when the number of external Legs or the number of loops increase.

Background

One major idea behind recent developments is to avoid Feynman diagrams:

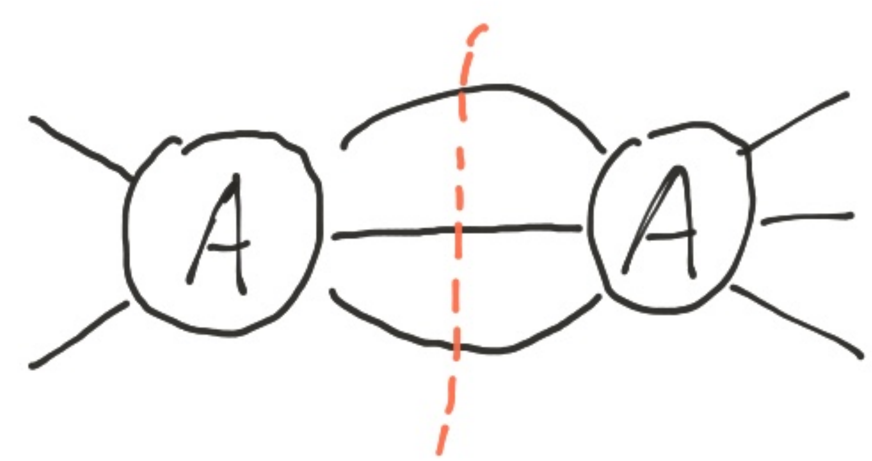
recursion relations, unitarity cut method, twistor space, Grassmannian and on-shell diagrams, bootstrap, CHY formulation, etc.

Unitarity Method

S-matrix is a Lorentz-Invariant analytic function of momentum variables with only those singularities required by **Unitarity**

— The S-Matrix program in 1960's.

Unitarity cuts:



$$\frac{1}{p_i^2} \to \delta^{(+)}(p_i^2)$$

A result is gauranteed to be physical if it passes all possible unitarity cut checks.

Goal :

Test colour-kinematics duality at 5-loop.

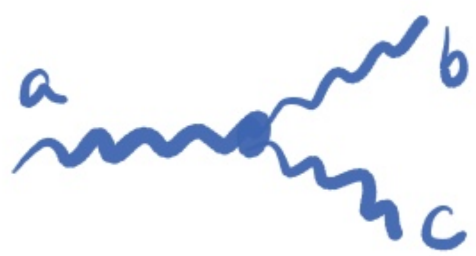
Construct 5-loop Sudakov form factor in $\mathcal{N}=4$ SYM via Colour-kinematics duality, together with unitarity checks.

What is colour-kinematics duality?

Colour-kinematics duality

colour factors $\longleftrightarrow$ **kinematic factors**

$\tilde{f}^{abc} = \mathrm{Tr}\left([T^a, T^b]T^c\right)$



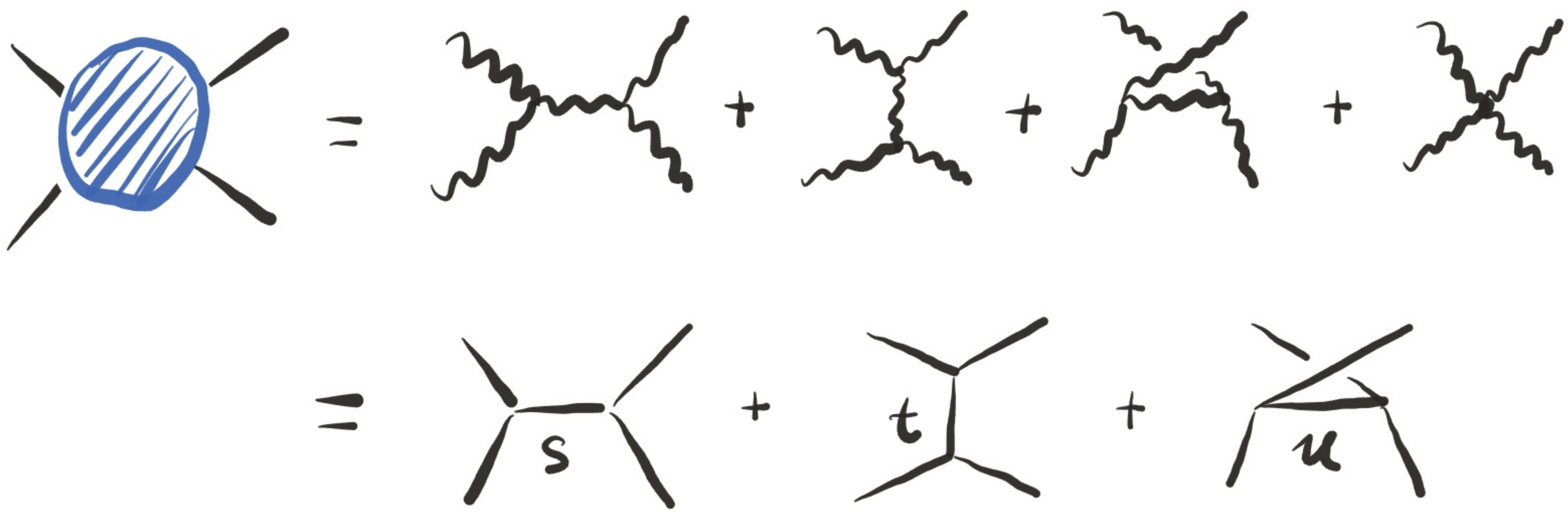
$s_{ij} = (p_i + p_j)^2$

More precisely, it conjectures that:

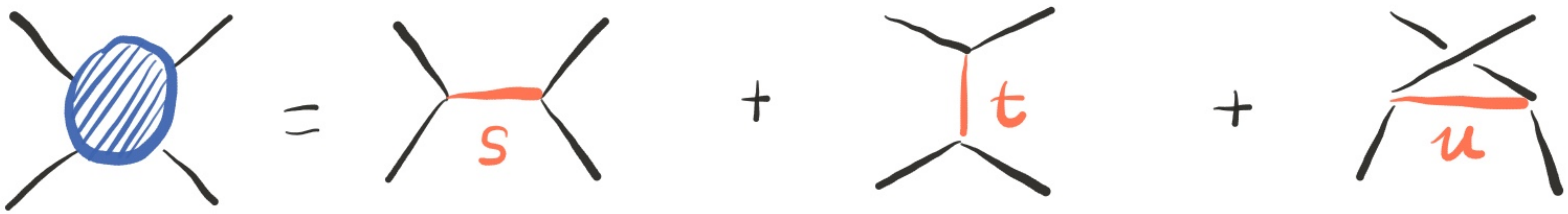
(1) there is a trivalent graph representation

(2) the kinematic numerators satisfy equations same as the Jacobi identity of the colour factors.

Example: four-point tree amplitude.



Example: four-point tree Amplitude.



$$= \frac{(f^{a_1a_2b}f^{ba_3a_4})\, n_s}{s} + \frac{(f^{a_4a_1b}f^{ba_2a_3})\, n_t}{t} + \frac{(f^{a_1a_3b}f^{ba_2a_4})\, n_u}{u}$$

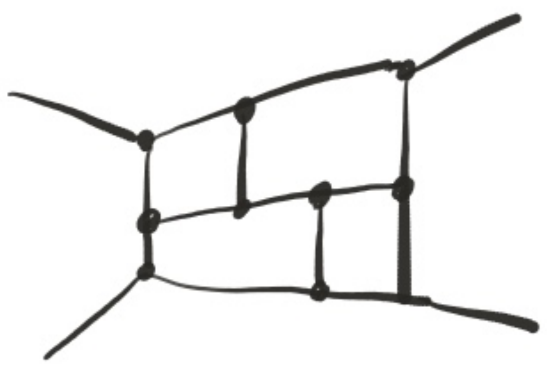
$$= \frac{c_s \cdot n_s}{s} + \frac{c_t \cdot n_t}{t} + \frac{c_u \cdot n_u}{u}$$

$$\boxed{c_s = c_t + c_u} \Longrightarrow \boxed{n_s = n_t + n_u}$$

Jacobi identity

Colour-kinematics duality @ Loops [Bern, Carrasco, Johansson]

$$\sum_{\text{trivalent graphs}}$$



$$= \sum_{\text{trivalent graphs}} \int \prod_{i=1}^{L} d^D l_i \times \frac{C_\alpha \cdot n_\alpha}{\prod (\text{propagator})} .$$

$$C_\alpha = \prod f^{abc} \qquad n_\alpha = \text{poly}(s_{ij})$$

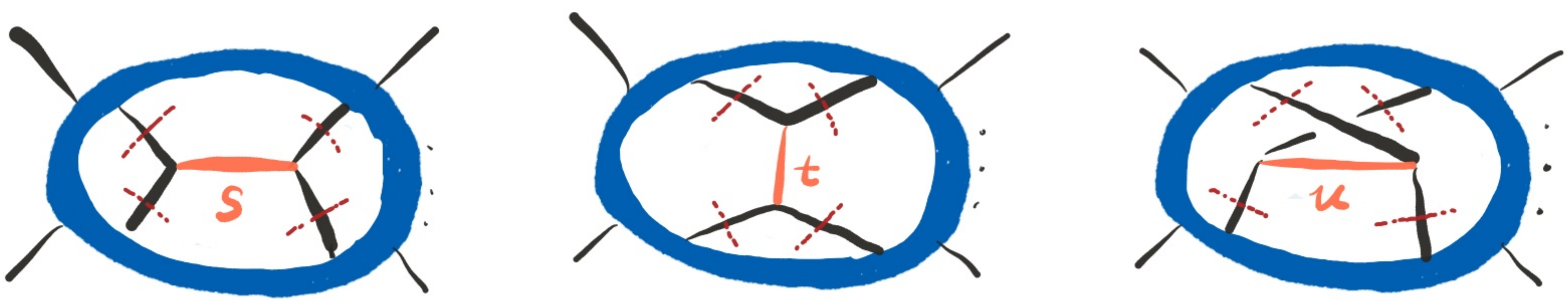
C-K duality askes for that : [CONJECTURE]

$$\boxed{C_\alpha = C_\beta + C_\gamma} \Rightarrow \boxed{n_\alpha = n_\beta + n_\gamma}$$

strong constraints on n_α!

Colour-kinematics duality @ Loops

This can be understood if we cut around a propagator



The Non-trivial point of the conjecture is that the duality holds without any cut.

Double copy & Gravity

[Bern, Carrasco, Johansson]
[Kawai, Lewellen, Tye]
See also He's talk.

YM × YM $\Rightarrow$ Gravity

$$c_\alpha \cdot n_\alpha \qquad\qquad \tilde{n}_\alpha \cdot n_\alpha$$

Get gravity amplitudes from YM for free, once a C-K dual representation is found.

Lots of work on this.

Interesting work on classical solutions [Monteiro, O'Connell, White]

This will not be considered in this talk

Success up to four-loop

[Bern, Carrasco, Dixon, Johansson, Roiban]

[Boels, Kniehl, Tarasov, GY]

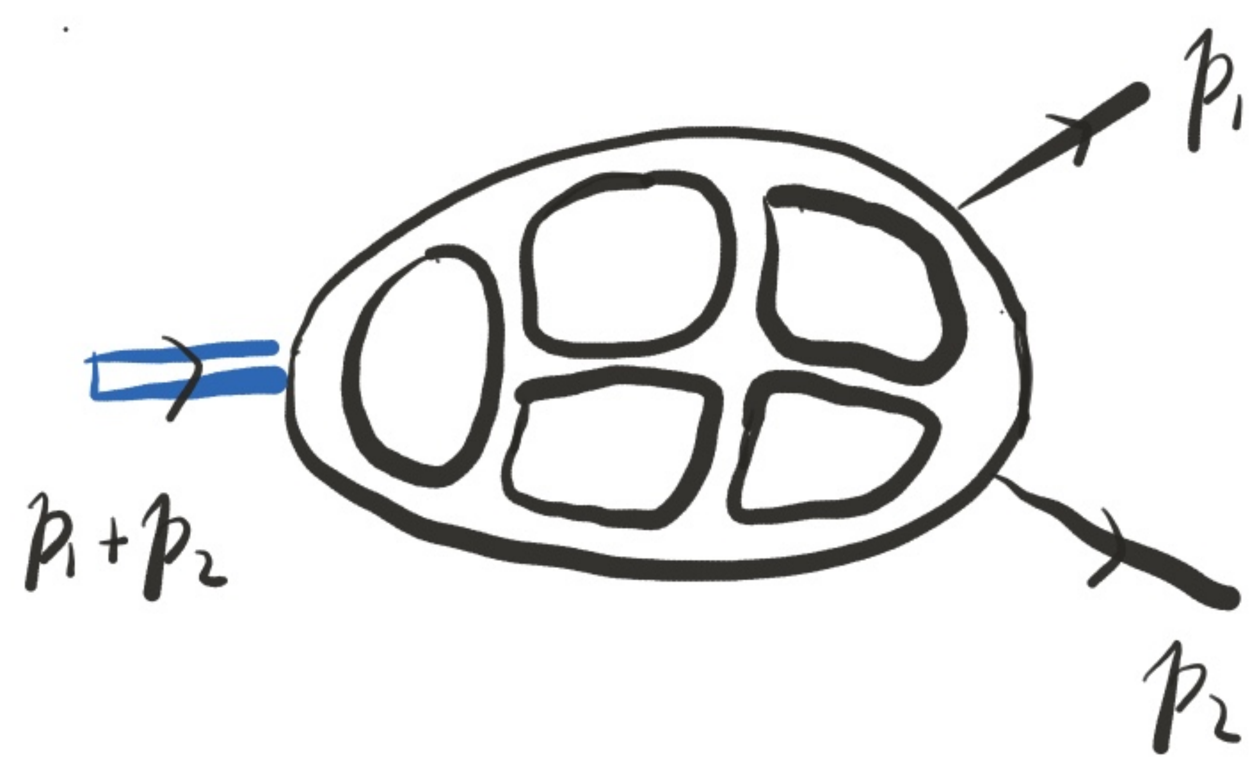
However, No five-loop solution satisfying Colour-kinematics duality had yet been found.

In the following, we will provide a five-loop solution.

Five-Loop construction

Our example:

Sudakov form factor in $\mathcal{N}=4$ SYM



General strategy

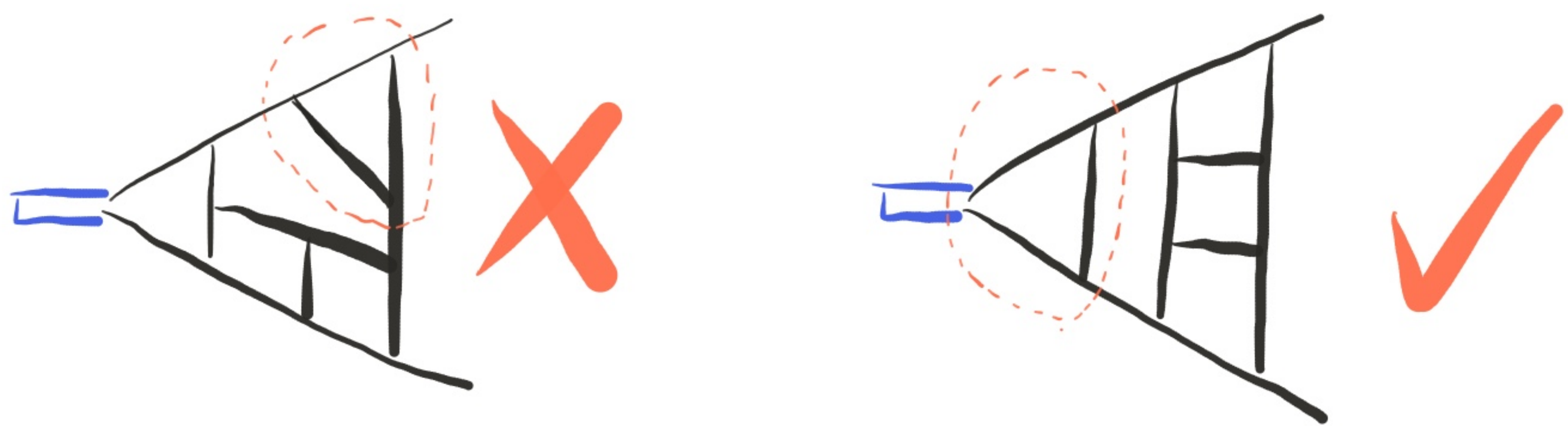
[Bern, Carrasco, Dixon, Johansson, Roiban]

[Boels, Kniehl, Tarasov, GY]

- Apply colour-kinematics duality to generate an **Ansatz**, which depends **linearly** on a set of parameters
- Fix parameters via symmetries, simple cuts
- Check and fix remaining parameters against **General Unitarity Cuts**.

Trivalent graphs

No one-loop bubble or triangle subgraph.



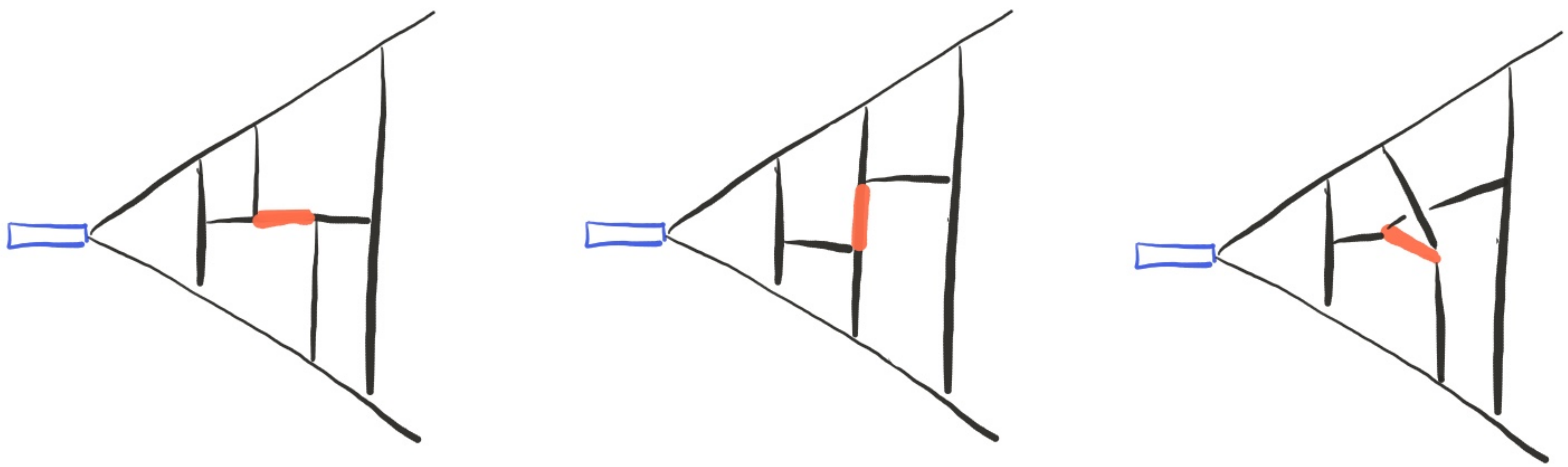
370 trivalent graphs to consider.

Trivalent graphs

Statistics for A_4 and F_{Sudakov}:

# of loops	1	2	3	4	5
# of top. for A_4	1	2	12	85	~1000
# of top. for F_{Sudakov}	1	2	6	37	370

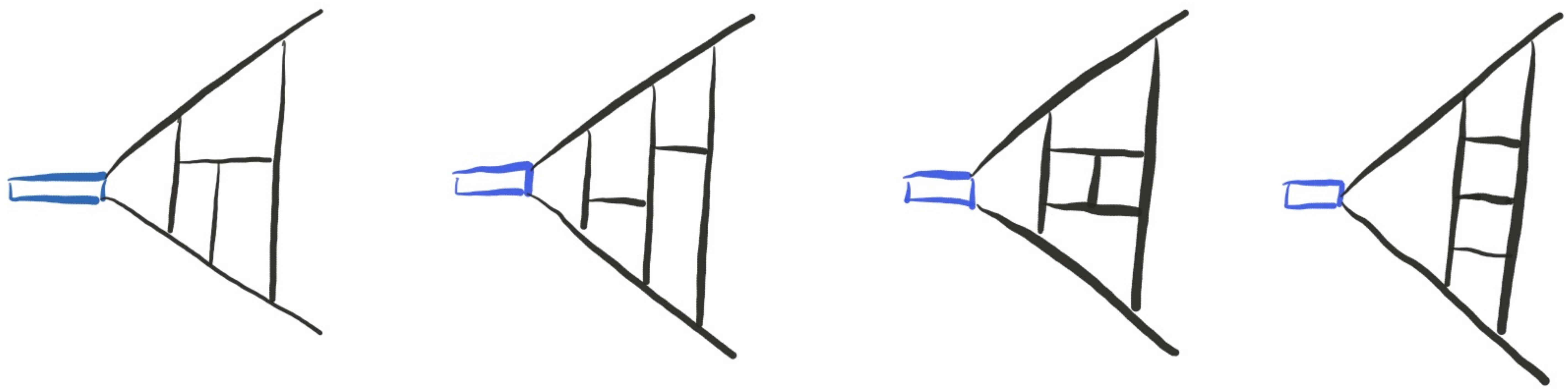
Jacobi relations



Each propagator contributes to such a relation

⇨ A huge set of relations between graphs and their numerators

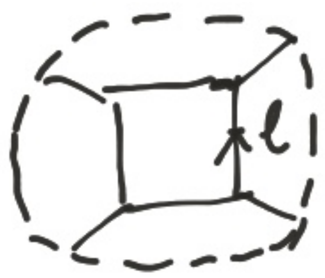
Master graphs



Numerators of all other graphs can be generated from these four graphs via Jacobi relations recursively.

Numerator Ansatz

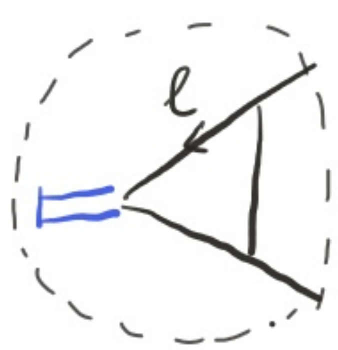
- dimension $[N_\alpha] = [p]^6$
- n-gon 1-loop subgraph carries at most $(n-4)$ powers of loop momenta



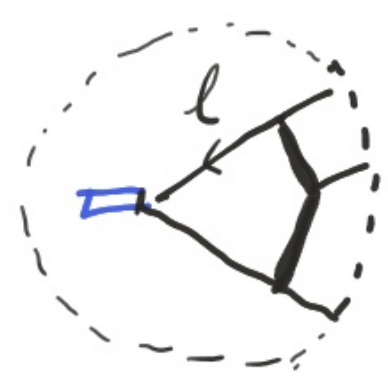
$\sim \ell^0$,

$\sim \ell \cdot p_i$,

- for n-gon subloop attached to the q-leg $\to (n-3)$ powers of loop mom.



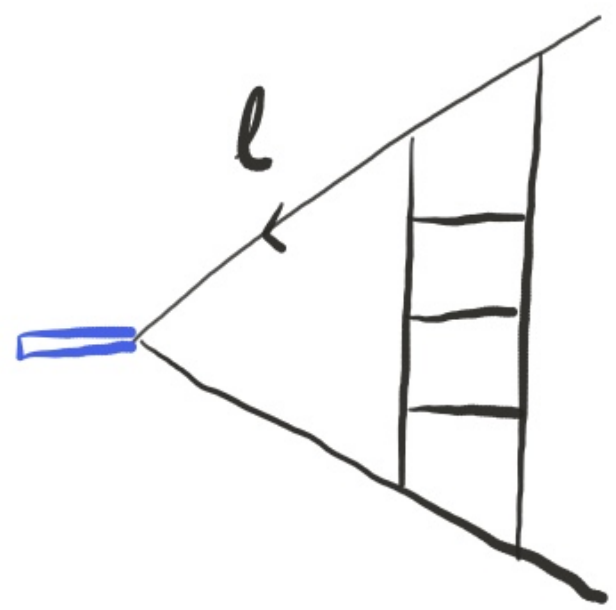
$\sim \ell^0$,



$\sim \ell \cdot p_i$

Numerator Ansatz

Example:



$$N^{\text{ansatz}} = \sum_{j=1}^{13} x_i M_i$$

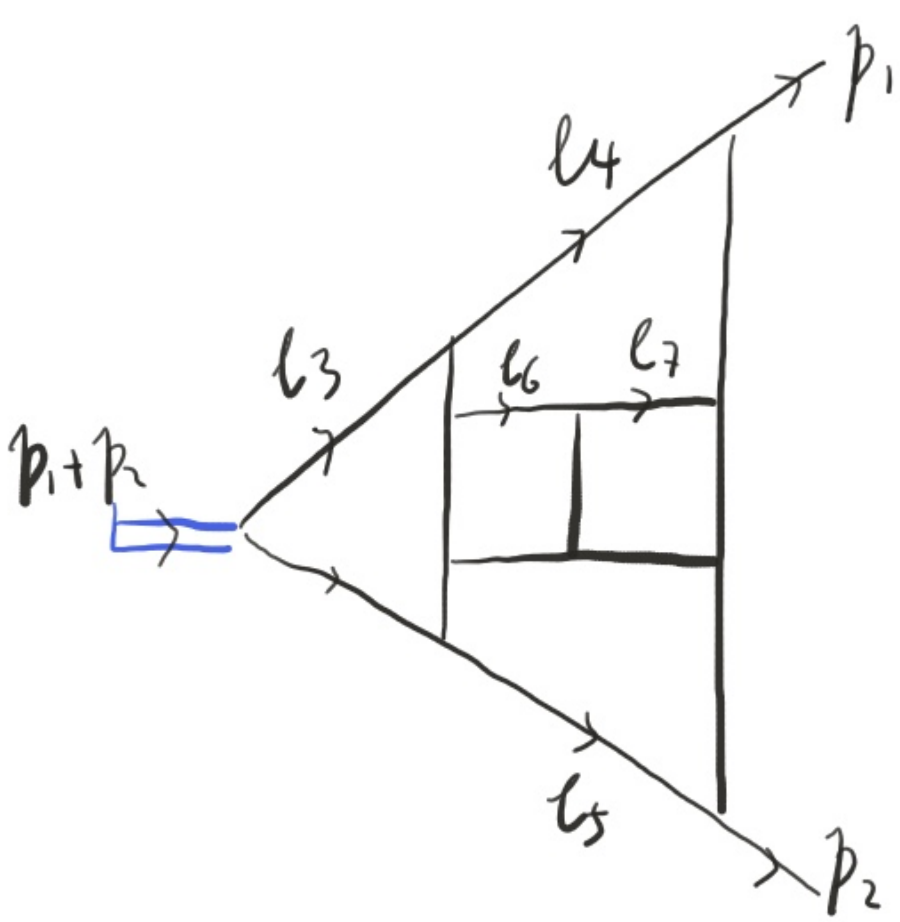
at most cubic in ℓ:

All possible monomials:

$$\Big\{ (\ell\cdot p_1)^2(\ell\cdot p_2),\ (\ell\cdot p_1)(\ell\cdot p_2)^2,\ (\ell\cdot p_1)^3,\ (\ell\cdot p_2)^3,$$
$$\ell^2(\ell\cdot p_1)s_{12},\ \ell^2(\ell\cdot p_2)s_{12},$$
$$(\ell\cdot p_1)^2 s_{12},\ (\ell\cdot p_2)^2 s_{12},\ (\ell\cdot p_1)(\ell\cdot p_2)s_{12}$$
$$(\ell\cdot p_1)s_{12}^2,\ (\ell\cdot p_2)s_{12}^2,\ \ell^2 s_{12}^2,\ s_{12}^3 \Big\} = M.$$

All 4 masters $\Rightarrow$ An ansatz of 162 parameters.

Fix parameters : Symmetries



Numerator should be be invariant under the symmetry transformation :

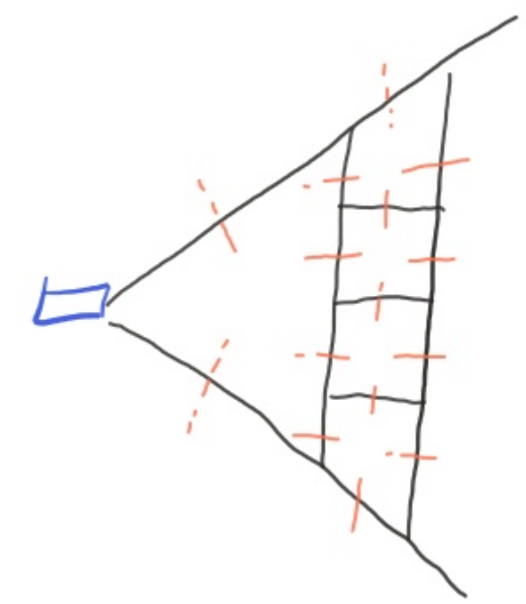
$$l_3 \to p_1+p_2-l_3$$
$$l_4 \to l_5$$
$$l_5 \to l_4$$
$$l_6 \to p_1+p_2-l_4-l_5-l_6$$
$$l_7 \to p_1+p_2-l_4-l_5-l_7$$

Symmetries of all trivalent topologies :

$\Rightarrow$ fix **115** parameters !

Fix parameters

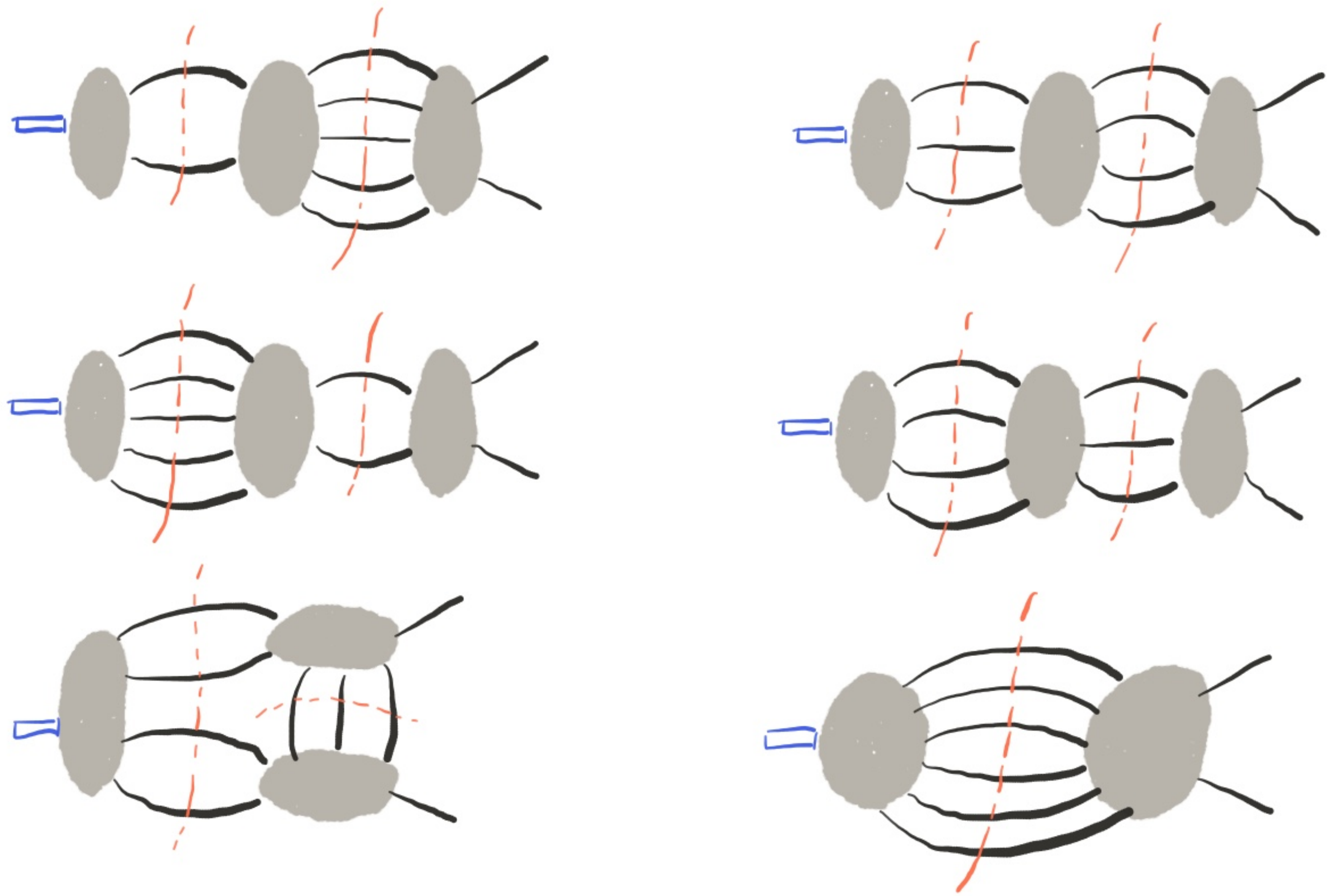
- Maximal cuts of master integrals further fix 27 parameters



- Require all Jacobi relations are satisfied fix 10 more parameters

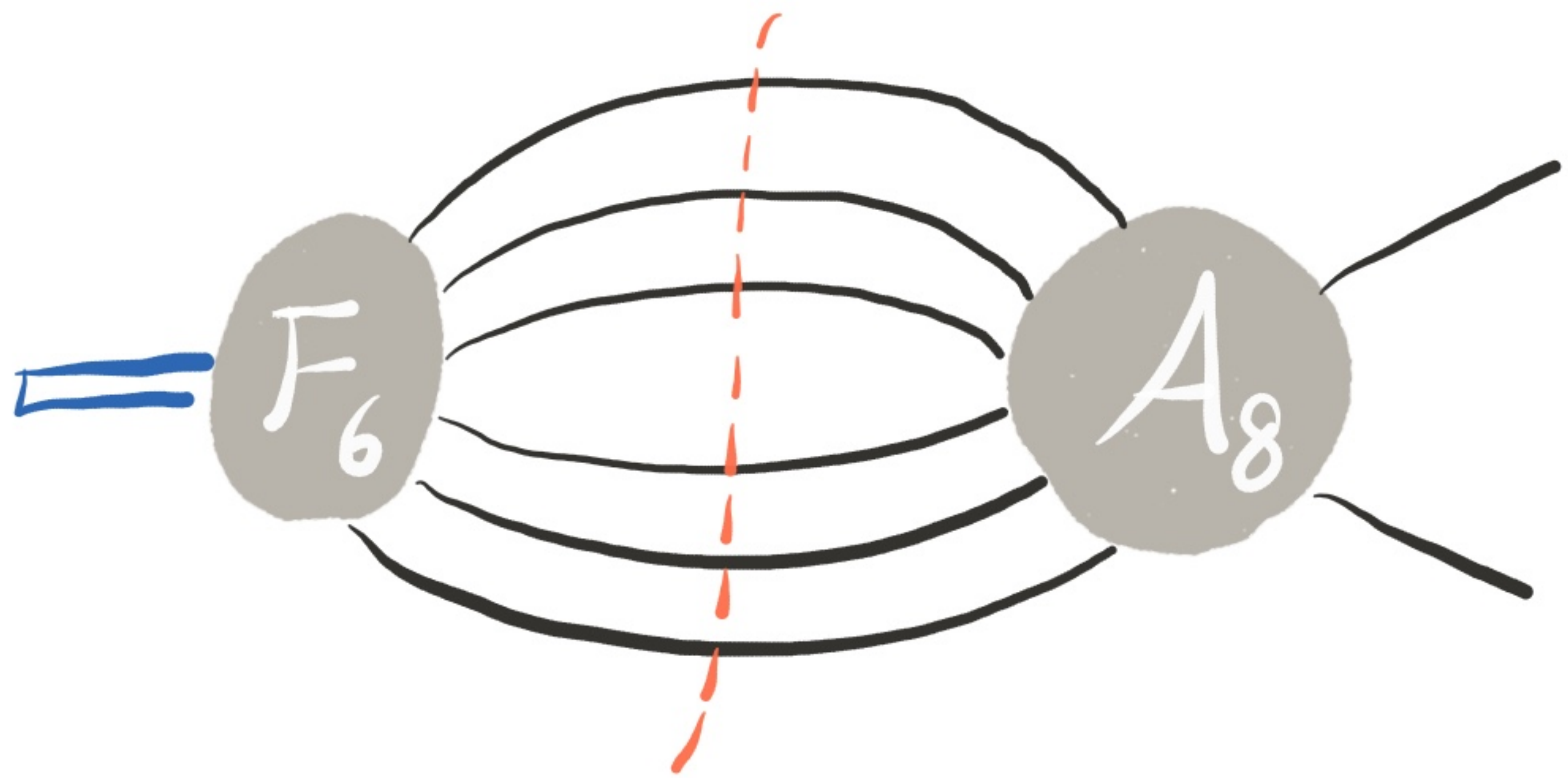
⇨ Now we are left with only 10 parameters!

Non-trivial Unitarity Cuts



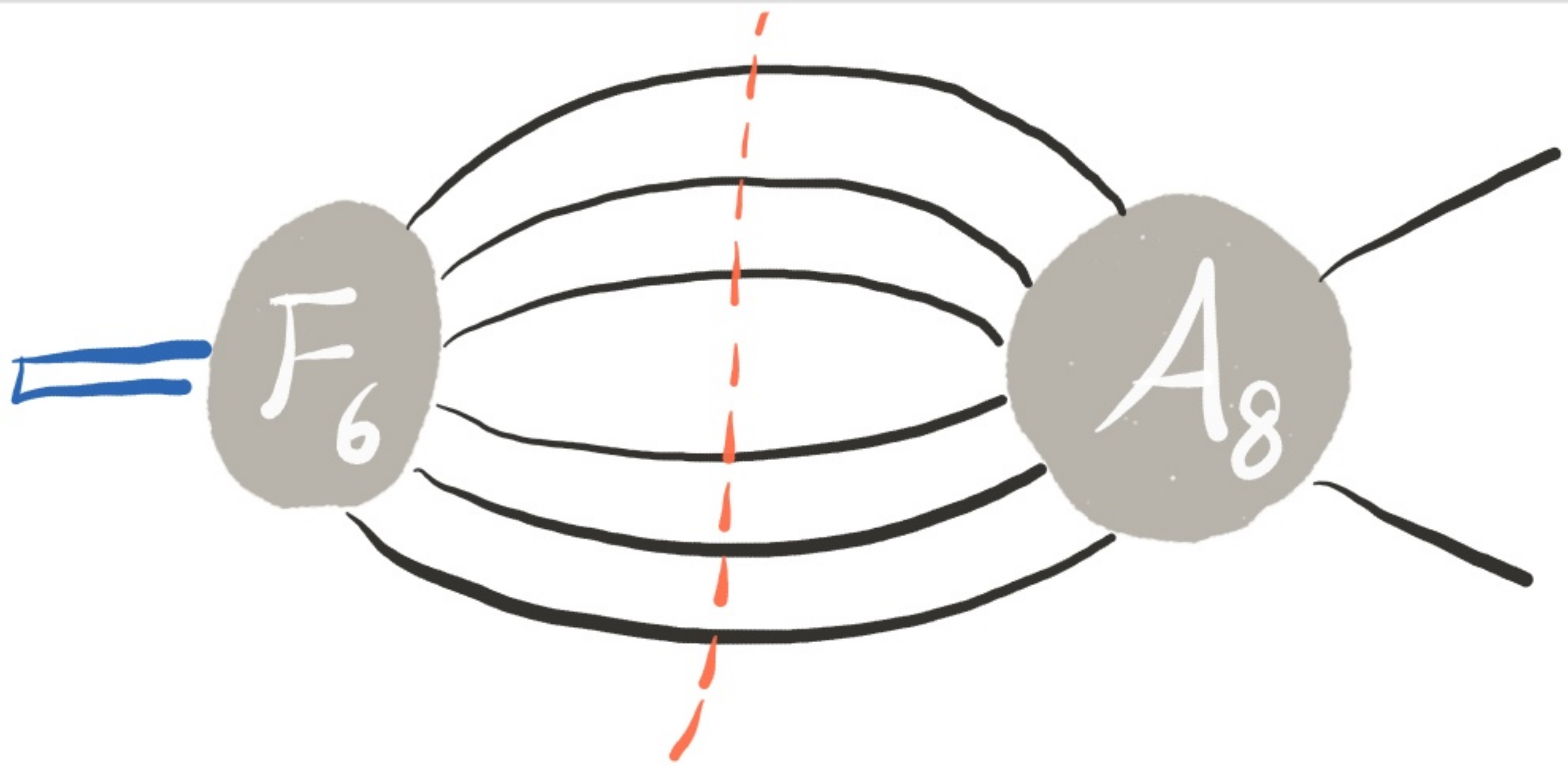
All satisfied + fix 7 parameters ⇒ A result with 3 parameters

Sextuple-cut



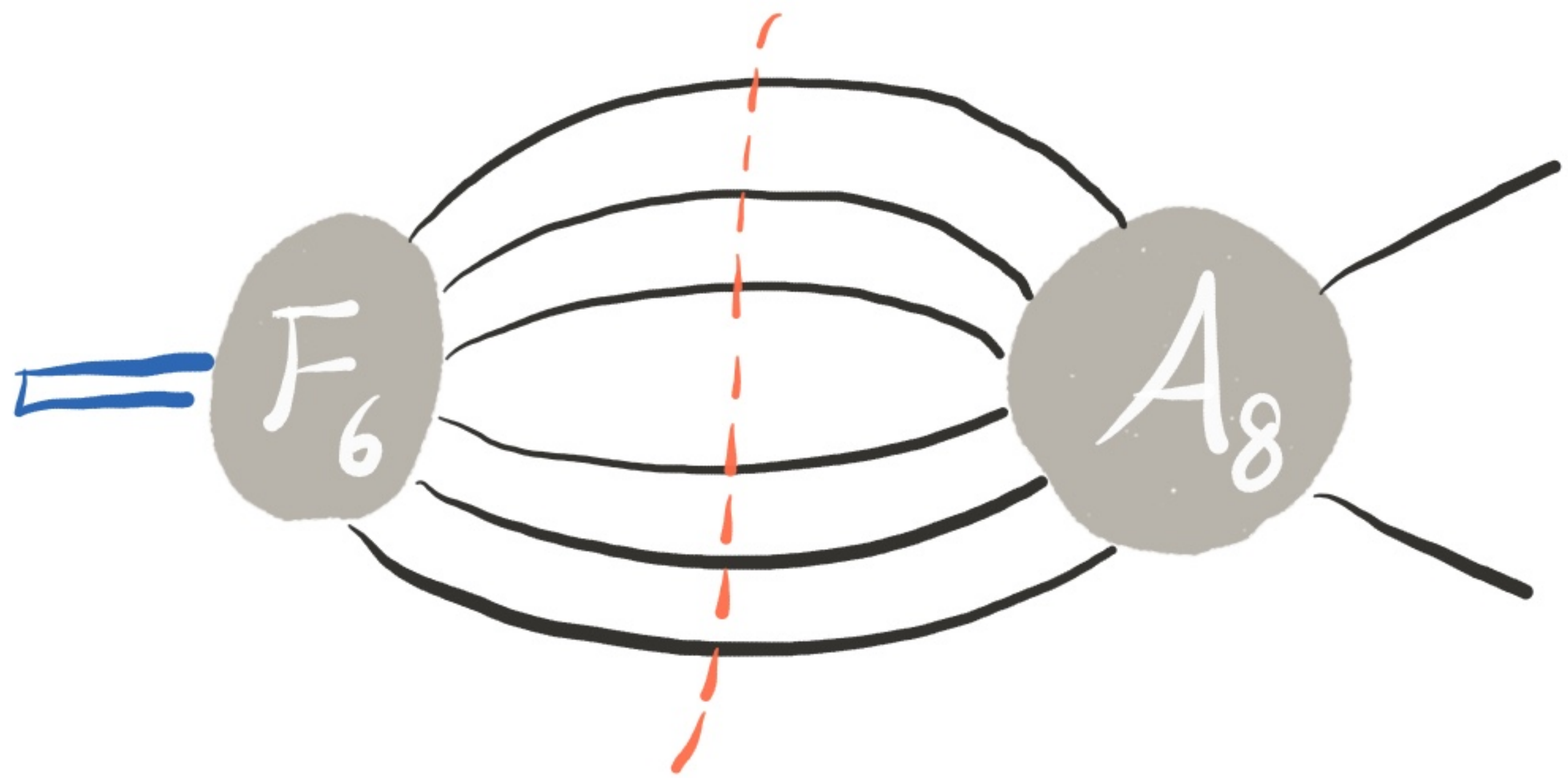
$$\sum_{\text{states}} F_6^{\text{tree}} \times A_8^{\text{tree}} \quad \text{v.s.} \quad \sum_{\substack{\text{trivalent}\\ \text{graphs}}} \underbrace{N_\alpha \cdot \Delta I_\alpha}_{\text{Ansatz}} \Big|_{\text{sextuple-cut}}$$

Sextuple-cut



$$\sum_{\text{states}} F_6^{\text{tree}} \times A_8^{\text{tree}} = \int \prod_{j=1}^{6} d^4\eta_{\ell_j} \Big\{ F_6^{\text{MHV}} \times A_8^{\text{N}^4\text{MHV}} + F_6^{\text{NMHV}} \times A_8^{\text{N}^3\text{MHV}} + F_6^{\text{N}^2\text{MHV}} \times A_8^{\text{N}^2\text{MHV}} + F_6^{\text{N}^3\text{MHV}} \times A_8^{\text{NMHV}} + F_6^{\text{N}^4\text{MHV}} \times A_8^{\text{MHV}} \Big\}$$

Sextuple-cut



$$\sum_{\substack{\text{trivalent}\\ \text{graphs}}} \sum_{\substack{\text{diff.}\\ \text{cuts}}} N_\alpha \cdot \Delta I_\alpha \Big|_{\text{sextuple-cut}}$$

~100 trivalent graphs ⇨ ~2,000 cut diagrams

This perfectly matches with tree products!

Conclusion & Outlook

Conclusion

- We provide a concrete example that colour-kinematics duality works at 5-loop.
- The 5-loop Sudakov form factor being constructed is an important physical observable by itself.

Outlook

- Other 5-loop examples?
- Sudakov form factor @ 6-loop?
- Double copy of form factor?

- Why does colour-kinematics duality work at all?

Thank You !